

Hall Conductivity near the $z=2$ Superconductor-Insulator Transition in 2D

Denis Dalidovich¹ and Philip Phillips^{2,3}

¹*National High Field Magnetic Laboratory,
Florida State University, Tallahassee, Florida, 32310*

²*Loomis Laboratory of Physics,
University of Illinois at Urbana-Champaign,
1100 W.Green St., Urbana, IL, 61801-3080*

³*James Franck Institute and Dept. of Physics, University of Chicago
5640 S. Ellis Ave., Chicago, Il. 60637*

We analyze here the behavior of the Hall conductivity σ_{xy} near a $z = 2$ insulator-superconductor quantum critical point in a perpendicular magnetic field. We show that the form of the conductivity is sensitive to the presence of dissipation η , and depends non-monotonically on H once η is weak enough. σ_{xy} passes through a maximum at $H \sim \eta T$ in the quantum critical regime, suggesting that the limits $H \rightarrow 0$ and $\eta \rightarrow 0$ do not commute.

Remarkable recent experiments [1,2] on the insulator-superconductor transition in thin films have explored the role of coupling the 2D electron gas to a ground plane. Such coupling not only provides a source of dissipation but also breaks particle-hole symmetry because the uniformly-distributed frustrating offset charges in the ground plane cannot be eliminated by Cooper pair tunneling. Mason and Kapitulnik [1] observed that this upgrade promotes superconductivity driving the system closer to phase coherence while at the same time the insulating state becomes more insulating. As it is known that dissipation [3] can diminish phase fluctuations, enhancement of superconductivity is expected as a result of coupling to a ground plane, as is seen experimentally. However, precisely how dissipation enhances the insulating nature of the insulating state is not known.

The issue of the insulating state aside, the inclusion of a ground plane can also be used to explore the role of particle-hole symmetry breaking. The obvious experimental probe of particle-hole symmetry breaking is the Hall conductivity. Only when such symmetry is broken does the Hall conductivity acquire a non-zero value. Experimentally [4], a non-zero Hall coefficient in thin films exhibiting the IST has never been observed. This suggests that in all extant experiments, particle-hole symmetry is present. Hence, it would be of utmost importance if the experimental set-up with a ground plane is used to measure the Hall coefficient. Such measurements would be instrumental in delineating how particle-hole asymmetry leads to a non-zero Hall coefficient. However, currently no theory exists for the Hall coefficient in the vicinity of the IST quantum critical point. It is the formulation of the Hall coefficient near the IST that we develop here.

The inclusion of particle-hole symmetry breaking results in a fundamental change of the dynamical exponent from $z = 1$ to $z = 2$. This can be seen immediately from the following argument. In a quantum rotor model, the

charging term in the presence of offset charges per rotor is of the form, $E_C(\hat{n}_i - n_0)^2$, where E_C is the charging energy and \hat{n}_i is the number operator per rotor. The offset charges enter through the constant term n_0 . In the corresponding action, the linear term in n_0 will be paired with a linear time derivative with respect to the phase, $\partial_\tau \theta$. As this term will provide the dominant frequency dependence, the time derivatives will count twice as much as will their spatial counterparts. Hence, $z = 2$. van Otterlo [5] et. al. have outlined an approach to calculate the transverse conductivity, σ_{xy} based on a Ginzburg-Landau (GL) approach. However, these authors did not include dissipation, either internal or external. In this brief note, we calculate the fluctuation Hall conductivity near the IST in the presence of dissipation, η . The particle-hole asymmetry is assumed to be strong, so that the system belongs to the $z = 2$ universality class. Transport in the absence of magnetic field was examined in Ref. ([6]) where it was shown that internal dissipation arising from the mutual scattering of quasiparticles is exponentially small at low temperatures. Hence, we expect that the dominant source of dissipation will arise from the coupling to a ground plane. We show that the fluctuation Hall conductivity is large once dissipation is weak enough, $\eta \ll 1$. In the quantum critical (QC) regime, for $H \ll \eta T$, the Hall conductivity is proportional to H and depends more singularly on η than the longitudinal one. For larger H the conductivity behaves as $1/H$, but is independent of η in agreement with the results of Ref. ([5]). This leads us to the conclusion that for $\eta \ll 1$ the dependence of the Hall conductivity on H is non-monotonic. We emphasize that we consider the fluctuation contribution to the Hall conductivity on the insulating side of the IST at low temperatures. In particular non-perturbative effects of a magnetic field are not included. Hence, we are not concerned here with the issue of the destruction of the superconducting phase by a finite magnetic field, in which case the relevant physics is

governed by dissipative motion of field-induced vortices [7].

The general form of the GL functional that models the behavior near the IST point in the presence of particle-hole asymmetry is [5]

$$F[\psi] = \int d^2r \int d\tau \left\{ \left[\left(\nabla + \frac{ie^*}{\hbar} \mathbf{A}(\mathbf{r}, \tau) \right) \psi^*(\mathbf{r}, \tau) \right] \cdot \left[\left(\nabla - \frac{ie^*}{\hbar} \mathbf{A}(\mathbf{r}, \tau) \right) \psi(\mathbf{r}, \tau) \right] + \lambda \psi^*(\mathbf{r}, \tau) \partial_\tau \psi(\mathbf{r}, \tau) + \kappa^2 |\partial_\tau \psi(\mathbf{r}, \tau)|^2 + \delta |\psi(\mathbf{r}, \tau)|^2 + \frac{u}{2} |\psi(\mathbf{r}, \tau)|^4 \right\} + F_{\text{dis}} \quad (1)$$

where $\mathbf{A}(\mathbf{r}, \tau)$ is the vector potential due to the applied electric field, $e^* = 2e$, and δ is proportional to the inverse correlation length. In Fourier space, the dissipation term, $F_{\text{dis}} = \eta \sum_{\mathbf{k}, \omega_n} |\omega_n| |\psi(\mathbf{k}, \omega_n)|^2$ corresponds to the Ohmic model of Caldeira and Leggett [8,9]. The parameters κ and λ measure the strength of quantum fluctuations. We will regard λ to be on the order of unity. Consequently, the term proportional to κ is irrelevant, and all parameters having the dimensionality of energy can be measured in units of λ . The $z = 2$ universality class renders the quartic interaction in Eq. (1) marginally irrelevant, making it possible to perform all calculations in the critical region with logarithmic accuracy [6,10]. In two dimensions, the static Hall conductivity obeys the scaling relation [11]

$$\sigma_{xy}(\delta, T, H, u) = \sigma_{xy}(T(l^*), H(l^*), u(l^*)). \quad (2)$$

In the momentum-shell RG, l^* is the scale at which the effective size of the Kadanoff cell becomes on the order of the correlation length. The magnetic field scales trivially as $H(l) = He^{zl}$ and we will assume it to be weak enough. This means that at the point when the scaling stops, $\delta(l^*) = 1$ and $H(l^*) \ll 1$. This allows us to neglect the discreteness of the Landau energy levels and to obtain the same one-loop RG equations as in the absence of a magnetic field,

$$\frac{d\delta(l)}{dl} = 2\delta(l) + f^{(2)}(\delta(l), T(l))u(l) \quad (3)$$

and

$$\frac{du(l)}{dl} = -f^{(4)}(\delta(l), T(l))u(l)^2, \quad (4)$$

where $f^{(2)}$ and $f^{(4)}$ are some complicated functions that depend strongly on the relation between λ and η . In the leading order, however, the scaled parameters $H(l^*)$, $T(l^*)$ and $u(l^*)$ are insensitive to the form of the particular functional form [6]. Hence, we obtain within logarithmic accuracy that in the QD regime, $l^* = \frac{1}{2} \ln(1/\delta)$, giving

$$T^* = \frac{T}{\delta}, \quad H^* = \frac{H}{\delta}. \quad (5)$$

In the QC regime, one requires a double logarithmic accuracy to obtain the leading order, so that $l^* = \frac{1}{2} \ln(\frac{1}{T} \ln \frac{1}{T})$, while

$$T^* = \ln \ln \frac{1}{T}, \quad H^* = \frac{H}{T} \ln \ln \frac{1}{T}. \quad (6)$$

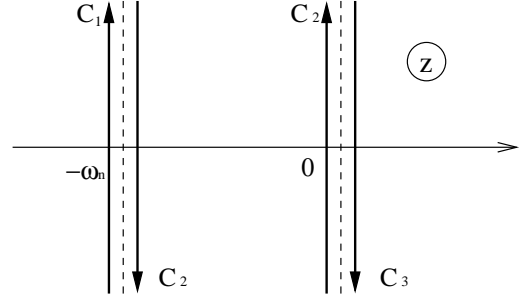


FIG. 1. The contour of integration in the complex plane z . The cuts are drawn along the imaginary axis at $z = -\omega_n$ and $z = 0$.

Sufficiently close to the critical point, the interaction scales to zero. Hence, we can calculate the conductivity with the help of the Kubo formula,

$$\sigma_{xy}(i\omega_n) = -\frac{\hbar}{\omega_n} \int d^2r \int d\tau \frac{\delta^2 \ln Z}{\delta \mathbf{A}_x(\tau, \mathbf{r}) \delta \mathbf{A}_y(\mathbf{0})} e^{i\omega_n \tau},$$

applied to the Gaussian part of Eq. (1) only. A simple calculation leads to the result

$$\begin{aligned} \sigma_{xy}(i\omega_\nu^*) &= \frac{i(e^*)^2}{2\hbar\omega_\nu^*} T^* (H^*)^2 \sum_{\omega_m^*} \sum_{n=0}^{\infty} (n+1) \\ &\times [G(\omega_m^* + \omega_\nu^*, n+1) G(\omega_m^*, n) \\ &- G(\omega_m^*, n+1) G(\omega_m^* + \omega_\nu^*, n)], \end{aligned} \quad (7)$$

where $G(\omega_m^*, n) = (i\lambda\omega_m^* + \eta|\omega_m^*| + \epsilon_n^*)^{-1}$ is the usual Matsubara Green function. The rescaled temperature T^* and the energy of quasiparticles $\epsilon_n^* = 1 + H^*n$ are employed in the right-hand side in accordance with Eq. (2) ($\omega_m^* = 2\pi m T^*$). We must perform then an analytical continuation to real frequencies after doing the summation over ω_m^* . The latter can be performed by transforming the sum over ω_m^* into the integral over the contour in the complex plane, shown on Fig. (1). The subsequent expansion over the small external frequency $\omega = -i\omega_\nu$ yields

$$\begin{aligned} \sigma_{xy} &= \frac{i(e^*)^2}{8\pi\hbar} (H^*)^2 \sum_{n=0}^{\infty} (n+1) \int_{-\infty}^{\infty} \coth \frac{z}{2T^*} \\ &\times \left\{ [G_n^R(z) - G_n^A(z)] \left[\frac{\partial G_{n+1}^R(z)}{\partial z} + \frac{\partial G_{n+1}^A(z)}{\partial z} \right] \right. \\ &\left. - [G_{n+1}^R(z) - G_{n+1}^A(z)] \left[\frac{\partial G_n^R(z)}{\partial z} + \frac{\partial G_n^A(z)}{\partial z} \right] \right\}, \end{aligned} \quad (8)$$

where the retarded and advanced Green functions

$$G_n^{R/A} = \frac{1}{\lambda z + \epsilon_n^* \mp i\eta z} \quad (9)$$

have been introduced.

It is evident that the structure of the Hall conductivity is different from its longitudinal counterpart. In fact, it is not entirely transparent how the various asymptotic forms can be extracted. The situation simplifies, however, for the case of weak magnetic fields $H^* \ll 1$, the case of interest here. When $H^* \ll \eta$, one can approximate

$$G_{n+1}^{R/A}(z) - G_n^{R/A}(z) = -H^*[G_n^{R/A}(z)]^2, \quad (10)$$

and show that the bracketed expression in Eq. (8) reduces to $-\frac{1}{3} \frac{\partial}{\partial z} [G_n^R(z) - G_n^A(z)]^3$. Switching then from summation over n to the integration over $t = H^*n$ and integrating subsequently by parts over z , we obtain ($z = 2T^*x$)

$$\sigma_{xy} = -\frac{8(e^*)^2}{3\pi h} \eta^3 (T^*)^3 H^* \int_0^\infty t dt \int_{-\infty}^\infty \frac{x^3 dx}{\sinh^2 x} \times \frac{1}{[(1+t+2\lambda T^*x)^2 + 4(\eta T^*x)^2]^3}. \quad (11)$$

From this expression it is immediately obvious that when $\lambda = 0$ so that particle-hole symmetry is reinstated, the integrand is an odd function of x and hence vanishes identically when integrated over the even limits. This result is expected because particle-hole asymmetry is essential for the Hall conductivity to be non-zero.

Quantum disordered regime. In this regime $T^* \ll 1$. For weak dissipation, $\eta \ll 1$ [6,12], the main contributions to the integral over x come from the region near $x = 0$ and from the vicinity of $x_0 = -(1+t)/2\lambda T^*$. To obtain the first contribution, $\sigma_{xy}^{(1)}$, we expand the integrand in Eq. (11) for small x . Performing then simple integrations and using Eq. (5), we find that

$$\sigma_{xy}^{(1)} = \frac{128\pi^3 e^2 \lambda \eta^3 H T^4}{225 h \delta^5}. \quad (12)$$

To calculate the second contribution, we expand the denominator of the integrand around x_0 to arrive at the result,

$$\sigma_{xy}^{(2)} = \frac{e^2 \lambda^3 H T}{h \eta^2 \delta^2} e^{-\delta/\lambda T}, \quad H \ll \eta \delta. \quad (13)$$

The total conductivity for $\eta \ll 1$ in the QD regime can be approximately represented as $\sigma_{xy} = \sigma_{xy}^{(1)} + \sigma_{xy}^{(2)}$. The second contribution dominates only for very weak dissipation, while for $\eta \sim O(1)$, the Hall conductivity is given solely by Eq. (12).

The above derivation is correct assuming the condition $H^* \ll \eta$ holds for all x . Obviously, near x_0 the

expansion, Eq. (10), is not valid if $H \gg \eta$, affecting thus the calculation of $\sigma_{xy}^{(2)}$. In this case, we introduce $y = z + \epsilon_n^*/\lambda + H^*/2\lambda$ and calculate directly the difference of the Green functions using Eq.(9). Expanding then the cotangent in Eq. (8) for small y and H^* we obtain with sufficient accuracy, ($\eta \ll \lambda$)

$$\sigma_{xy}^{(2)} = \frac{4e^2 \eta^3 (H^*)^3}{\pi h \lambda^4 T^*} \sum_{n=0}^\infty (n+1) \frac{(\epsilon_n^*)^3}{\sinh^2(\epsilon_n^*/2\lambda T^*)} \times \int_{-\infty}^\infty \frac{y^2 dy}{\left[\left(y - \frac{H^*}{2} \right)^2 + \left(\frac{\eta \epsilon_n^*}{\lambda} \right)^2 \right]^2} \times \frac{1}{\left[\left(y + \frac{H^*}{2} \right)^2 + \left(\frac{\eta \epsilon_n^*}{\lambda} \right)^2 \right]^2}. \quad (14)$$

For $H^* \ll \eta$, one can neglect $H/2$ in the denominator of the equation above, and the resultant conductivity reduces to Eq. (13). In the opposite limit $H^* \gg \eta$, the contributions around the minima at $y = H^*/2$ and $y = -H^*/2$ should be calculated separately yielding

$$\sigma_{xy}^{(2)} = \frac{4e^2 \lambda T}{h H} e^{-\delta/\lambda T}, \quad H \gg \eta \delta. \quad (15)$$

This contribution is η -independent and inversely proportional to H , representing thus the $\eta \rightarrow 0$ collisionless limit that was obtained in Ref. ([5]).

Quantum critical regime. In this regime $T^* \gg 1$ with the double-logarithmic accuracy, and the entire contribution is determined by small z (y). We perform then analogously the integration over y in Eq. (14) and, using Eq. (6), obtain for the two limiting cases,

$$\sigma_{xy} = \frac{e^2}{h} \begin{cases} \frac{\lambda^3 H}{6\eta^2 T} \left(\ln \ln \frac{1}{T} \right)^2, & H \ll \eta T, \\ \frac{4\lambda}{H} \left(\ln \ln \frac{1}{T} \right) \left(\ln \ln \ln \frac{1}{T} \right), & H \gg \eta T. \end{cases} \quad (16)$$

The last result in the above formula is written with triple logarithmic accuracy.

We see that for small H , the conductivity is inversely proportional to η^2 , while for larger magnetic fields, it does not depend on dissipation at all. This corresponds to the existence of a finite collisionless limit for the static Hall conductivity. The results here are different from those for the longitudinal conductance that develops the Drude singularity once dissipation is neglected. In the collisionless limit, however, σ_{xy} is proportional to $1/H$ for all H , which has never been observed experimentally as $H \rightarrow 0$. Though we limited ourselves with the Gaussian approximation, the results obtained indicate that the limits $\eta = 0$, $H \rightarrow 0$ and $H = 0$, $\eta \rightarrow 0$ do not commute in the general scaling formula for the static conductivity near a quantum critical point,

$$\sigma_{xy} = (4e^2/h)\Sigma_{xy}(\eta/T, \delta^z/T, \sqrt{H}/\delta). \quad (17)$$

Our results suggest also that for weak dissipation, the dependence of σ_{xy} on H is non-monotonic, passing through a maximum at $H \sim \eta T$ in the QC regime. The non-monotonic dependence may be observed also in the QD regime with the maximum at $H \sim \eta\delta$. However, in this regime one requires η to be so weak that $\sigma_{xy}^{(1)}$ is always small, compared to $\sigma_{xy}^{(2)}$. Experimentally, the suggested dependences can be best tested in systems, in which the dissipation is the smallest energy scale. Such a situation might be realized in artificially fabricated JJA coupled to a ground plane. However, one should remember that the effects of magnetic frustration, neglected here, may affect the behavior of σ_{xy} for higher magnetic fields [13,14]. As a result the dependence of the Hall conductivity on magnetic field can reveal additional minima and maxima. Their origin, however, is not connected with dissipation, but rather a consequence of the flux quantization.

The calculations presented here are based on the action that describes also the fluctuations of the superconducting order parameter near a disorder tuned metal/d-wave superconductor transition [15]. However, η is usually of the order of unity in this case, and σ_{xy} considered here represents only the anomalous (fluctuation) part of the total conductivity. For $\eta \approx O(1)$ this anomalous part becomes dominant only unobservably close to the quantum critical point, in the region where $\ln \ln \frac{1}{T} \gg 1$. Otherwise, it is of the same order or smaller than the normal part and, hence, not interesting. The slow double logarithmic divergence of σ_{xy} obtained here, is in general agreement with the results obtained from the non-linear sigma model approach to the IST in a system of interacting bosons in the presence of disorder [16].

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